

Calculus: Look Before You Integrate

UTSA Problem Solving Club • Fall 2026

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Three Rational Integrals

Evaluate

$$(a) \int_0^1 \frac{dx}{1+x^2}, \quad (b) \int_0^1 \frac{x^2}{1+x^2} dx, \quad (c) \int_0^1 \frac{x^4}{1+x^2} dx.$$

One Reversal, Many Symmetries

Let $a < b$, let f be continuous on $[a, b]$, and let $m = (a + b)/2$ be the midpoint of the interval.

(a) Show that change of variables $u = (a + b) - x$ reverses the interval $[a, b]$, explain what happens to the graph of $y = f(x)$, and deduce that

$$\int_a^b f(x) dx = \int_a^b f(a + b - x) dx.$$

(b) How is the graph of a function f that satisfies the identity $f(a + b - x) = f(x)$? How about $f(a + b - x) = -f(x)$?

What can you say about the values of $\int_a^m f(x) dx$ (left half-interval $[a, m]$), $\int_m^b f(x) dx$ (right half-interval $[m, b]$) and $\int_a^b f(x) dx$ in each case?

Deduce the usual rules for the integrals of odd- and even-functions on a symmetric interval $[-c, c]$.

(c) If $f(x) + f(a + b - x) = C$ for a constant C , determine $\int_a^b f(x) dx$.

Two Fractions

Evaluate

$$(a) \int_0^1 \frac{x^2}{x^2 + (1-x)^2} dx, \quad (b) \int_2^4 \frac{e^x}{e^x + e^{6-x}} dx.$$

A Reciprocal Reversal

(a) Let $0 < a < b$ and let f be continuous on $[a, b]$. Prove that

$$\int_a^b \frac{f(x)}{1+x^2} dx = \int_{1/b}^{1/a} \frac{f(1/u)}{1+u^2} du.$$

(b) Evaluate $\int_{1/2}^2 \frac{\ln x}{1+x^2} dx$.

A Circle Away from the Origin

(a) Evaluate $\int_1^3 \sqrt{1-(x-2)^2} dx$ and $\int_1^3 x \sqrt{1-(x-2)^2} dx$.

(b) The disk $(x-2)^2 + z^2 \leq 1$ in the xz -plane is rotated about the z -axis. Find the volume of the resulting solid.

Is $\pi = 3.14\dots = 22/7$?

(a) Prove that

$$\int_0^1 \frac{x^4(1-x)^4}{1+x^2} dx = \frac{22}{7} - \pi.$$

(b) Use Simpson's rule to obtain a better approximation to the value of π than Archimedes' $22/7$.

(c) Use a calculator to compare the percentual approximation errors of $22/7$ vs. your new estimate of π .

Source: [Putnam 1968 A1](#) (part (a))

An Integral over a Half-Line

Evaluate

$$\int_0^\infty \frac{\arctan x}{1+x^2} dx.$$

Logarithms and Square Roots

Evaluate

$$\int_2^4 \frac{\sqrt{\ln(9-x)}}{\sqrt{\ln(9-x)} + \sqrt{\ln(x+3)}} dx.$$

Here $\ln$ denotes the natural logarithm.

Source: [Putnam 1987 B1](#)

A Function Given Indirectly

Find all functions $h : \mathbb{R} \setminus \{0, 1\} \rightarrow \mathbb{R}$ satisfying

$$h(x) + h\left(1 - \frac{1}{x}\right) = x \quad \text{for every } x \notin \{0, 1\}.$$

For such a function, evaluate $\int_2^3 h(x) dx$.

An Integral of an Unknown Function

Suppose that $f : \mathbb{R} \rightarrow \mathbb{R}$ satisfies

$$f(x) + f\left(1 - \frac{1}{x}\right) = \arctan x \quad \text{for all real } x \neq 0.$$

Find $\int_0^1 f(x) dx$.

Source: [Putnam 2016 A3](#)

A Solid Hidden in an Inequality

Find the volume of the region of points (x, y, z) such that

$$(x^2 + y^2 + z^2 + 8)^2 \leq 36(x^2 + y^2).$$

Source: [Putnam 2006 A1](#)